Theorem: 1

Properties of Continuous Functions

Let f and g be continuous functions on an interval I, let c be a real number and let n be a positive integer. The following functions are continuous on I.

1. Sums/Differences: $f \pm g$
2. Constant Multiples: $c \cdot f$
3. Products: $f \cdot g$
4. Quotients: f/g (as long as $g = 0$ on I)
5. Powers: f^n
6. Roots: $\sqrt[n]{f}$ (If n is even then require $f(x) \geq 0$ on I.)
7. Compositions: Adjust the definitions of f and g to: Let f be continuous on I, where the range of f on I is J and let g be continuous on J. Then $g \circ f$, i.e., $g(f(x))$, is continuous on I.

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Theorem: 2

Continuous Functions

Let n be a positive integer. The following functions are continuous on their domains.

1. $f(x) = \sin x$
2. $f(x) = \cos x$
3. $f(x) = \tan x$
4. $f(x) = \csc x$
5. $f(x) = \sec x$
6. $f(x) = \cot x$
7. $f(x) = a^x$ $(a > 0)$
8. $f(x) = \ln x$
9. $f(x) = \sqrt[n]{x}$

Example

Determining intervals on which a function is continuous

State the interval(s) on which each of the following functions is continuous

1. $f(x) = \sqrt{x-1} + \sqrt{5-x}$

The square root terms are continuous on the intervals $[1, \infty)$ and $(-\infty, 5]$, respectively. As f is continuous only where each term is continuous, f is continuous on $[1, 5]$.

2. $f(x) = x \sin x$

The function $y = x$ and $y = \sin x$ are each continuous everywhere, hence their product is, too.

3. $f(x) = \tan x$

Domain of $\tan x$ includes all real numbers except odd multiples of $\frac{\pi}{2}$. Thus the intervals on which $f(x) = \tan x$

4. $f(x) = \sqrt{\ln x}$

The domain of $y = \sqrt{x}$ is $[0, \infty)$. The range of $y = \ln x$ is $(-\infty, \infty)$, but if we restrict its domain to $[1, \infty)$ its range is $[0, \infty)$. So restricting $y = \ln x$ to domain of $[1, \infty)$ restricts its output is $[0, \infty)$, on which $y = \sqrt{x}$ is defined. Thus the domain of $f(x) = \sqrt{\ln x}$ is $[1, \infty)$.

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